

# Gauge invariant nonlocal mass operator in YM theories

*D. Dudal<sup>1</sup>, N. Vandersickel, H. Verschelde*

*Ghent University, Department of Mathematical Physics and Astronomy, Krijgslaan 281 S9, Belgium*

*J.A. Gracey*

*Theoretical Physics Division, Department of Mathematical Sciences, University of Liverpool, P.O. Box 147, Liverpool, L69 3BX, United Kingdom*

*M.A.L. Capri, V.E.R. Lemes, S.P. Sorella*

*Instituto de Física, UERJ, Universidade do Estado do Rio de Janeiro, Rua São Francisco Xavier 524, 20550-013 Maracanã, Rio de Janeiro, Brasil*

*R.F. Sobreiro*

*CBPF, Centro Brasileiro de Pesquisas Físicas, Rua Xavier Sigaud 150, 22290-180, Urca, Rio de Janeiro, Brasil*

## 1 Introduction

Lattice simulations in the Landau or maximal Abelian gauges have made clear the need for mass parameters in the fitting functions [1, 2, 3]. These fits involve e.g. a Yukawa propagator  $\sim \frac{1}{q^2 + M^2}$ . Moreover, phenomenological studies frequently make use of an effective gluon mass [4]. Theoretical studies, based on Schwinger-Dyson equations and the Pinch Technique report a dynamical gluon mass [5, 6, 7]. Also alternative analytical calculations gave such evidence, see e.g. [8]. Finally, let us mention the issue of  $\frac{1}{q^2}$  power corrections in physical correlators, tackled with QCD sum rules [9], lattice and OPE techniques [10] or even via the AdS/QCD picture [11]. A natural question arising is where does such a mass scale originate from? We recall that the standard Yang-Mills action cannot contain a mass due to gauge invariance. The Higgs mechanism is not a solution here, due to the associated gauge symmetry breaking. A natural answer is that a dynamical mass scale is generated by nonperturbative effects, in the form of a dimension 2 condensate. Of course, then the question pops up which  $d = 2$  operator  $\mathcal{O}$  to consider? In our opinion, a few requirements are to be met:  $\mathcal{O}$  should be gauge invariant, as it is supposed to enter physical

---

<sup>1</sup>david.dudal@ugent.be

quantities. It should be local, as nonlocal actions are hard to handle/interpret, and it should be renormalizable, as we want to perform quantum calculations, thus we want consistent finite results, renormalization group (RG) invariance, ... Zakharov et al proposed to use  $A_{\min}^2 = (VT)^{-1} \min_{U \in SU(N)} \int d^4x (A_\mu^U)^2$  which is gauge invariant but nonlocal [12]. Only in the Landau gauge do we have  $\langle A_{\min}^2 \rangle = A^2$  which is a renormalizable and condensing local composite operator, giving rise to effective dynamical gluon mass [8, 13]. We could also use the mass operator based on the transverse gluon field:  $(A_\mu^T)^2 = \left[ \left( \delta_{\mu\nu} - \frac{\partial_\mu \partial_\nu}{\partial^2} \right) A_\nu \right]^2$ , or the non-Abelian Stueckelberg term,  $\mathcal{O}_S = \left( A_\mu - \frac{i}{g} U^{-1} \partial_\mu U \right)^2$ ,  $U = e^{ig\phi^a T^a}$ . The previous proposals are all classically equivalent [14], but face problems with nonlocality, nonpolynomiality, nonrenormalizability, ... We proposed to study  $\mathcal{O} = \int d^4x F_{\mu\nu}^a \left[ (D_\sigma^2)^{-1} \right]^{ab} F_{\mu\nu}^b$  [14], which we couple to the Euclidean YM action via

$$S_{\mathcal{O}} = -\frac{m^2}{4} \int d^4x F_{\mu\nu}^a \left[ (D_\sigma^2)^{-1} \right]^{ab} F_{\mu\nu}^b, \quad (1)$$

where  $D_\sigma$  denotes the covariant derivative. This operator  $\mathcal{O}$  already found use in 3D YM [15]. This operator is clearly gauge invariant. Furthermore, it can be put quite easily in a local form, since we can replace  $S_{\mathcal{O}}$  with

$$S'_{\mathcal{O}} = \frac{1}{4} \int d^4x \bar{B}_{\mu\nu}^a D_\sigma^{ab} D_\sigma^{bc} B_{\mu\nu}^c + \frac{1}{4} \int d^4x \bar{G}_{\mu\nu}^a D_\sigma^{ab} D_\sigma^{bc} G_{\mu\nu}^c + \frac{im}{4} \int d^4x (B - \bar{B})_{\mu\nu}^a F_{\mu\nu}^a, \quad (2)$$

where  $B_{\mu\nu}^a, \bar{B}_{\mu\nu}^a$  are antisymmetric bosonic and  $G_{\mu\nu}^a, \bar{G}_{\mu\nu}^a$  fermionic (ghost) fields in the adjoint representation. Notice that for  $m = 0$ , we have in fact introduced a unity.

## 2 Renormalization analysis

We continue our investigation from the starting action

$$\begin{aligned} S &= \underbrace{\frac{1}{4} \int d^4x F_{\mu\nu}^a F_{\mu\nu}^a}_{S_{YM}} + \underbrace{\int d^4x \left( \frac{\alpha}{2} b^a b^a + b^a \partial_\mu A_\mu^a + \bar{c}^a \partial_\mu D_\mu^{ab} c^b \right)}_{S_{gf}} + \frac{1}{4} \int d^4x \bar{B}_{\mu\nu}^a D_\sigma^{ab} D_\sigma^{bc} B_{\mu\nu}^c \\ &+ \frac{1}{4} \int d^4x \bar{G}_{\mu\nu}^a D_\sigma^{ab} D_\sigma^{bc} G_{\mu\nu}^c + \frac{im}{4} \int d^4x (B - \bar{B})_{\mu\nu}^a F_{\mu\nu}^a. \end{aligned} \quad (3)$$

As the reader will notice, we imposed a linear gauge fixing, encoded in  $S_{gf}$ . The complete action enjoys a nilpotent  $BRST_1$  symmetry, generated by

$$\begin{aligned} s_1 A_\mu^a &= -D_\mu^{ab} c^b, \quad s_1 c^a = \frac{g}{2} f^{abc} c^b c^c, \quad s_1 B_{\mu\nu}^a = g f^{abc} c^b B_{\mu\nu}^c, \quad s_1 \bar{B}_{\mu\nu}^a = g f^{abc} c^b \bar{B}_{\mu\nu}^c, \\ s_1 G_{\mu\nu}^a &= g f^{abc} c^b G_{\mu\nu}^c, \quad s_1 \bar{G}_{\mu\nu}^a = g f^{abc} c^b \bar{G}_{\mu\nu}^c, \quad s_1 \bar{c}^a = b^a, \quad s_1 b^a = 0, \quad s_1^2 = 0. \end{aligned} \quad (4)$$

For  $m = 0$ , we can identify a nilpotent ‘‘supersymmetry’’  $\delta_s$  [16],

$$\delta_s B_{\mu\nu}^a = G_{\mu\nu}^a, \quad \delta_s \bar{G}_{\mu\nu}^a = \bar{B}_{\mu\nu}^a, \quad \delta_s \Psi = 0 \text{ for all other fields } \Psi, \quad \delta_s^2 = 0, \quad (5)$$

which can be invoked to establish another nilpotent  $BRST_2$  invariance for  $m = 0$ ,

$$s_2 = s_1 + \delta_s, \quad s_2^2 = 0. \quad (6)$$

For  $m \neq 0$ , we can embed the action (3) into a “larger” model with external sources [14]. In a particular physical limit, the original model is recovered. The reason why we introduced the “larger” model is because it simplifies the renormalizability analysis. If we can prove the renormalizability of this action, then also the physically relevant action will be renormalizable as a special case. The extended model obeys many Ward identities, including a Slavnov-Taylor identity. We constructed the most general action compatible with these Ward identities, and when the smoke cleared after taking the physical limit, we arrived at a renormalizable local action with mass terms [16],

$$\begin{aligned} S_{phys} &= S_{cl} + S_{gf}, \\ S_{cl} &= \int d^4x \left[ \frac{1}{4} F_{\mu\nu}^a F_{\mu\nu}^a + \frac{im}{4} (B - \bar{B})_{\mu\nu}^a F_{\mu\nu}^a \right. \\ &\quad + \frac{1}{4} (\bar{B}_{\mu\nu}^a D_\sigma^{ab} D_\sigma^{bc} B_{\mu\nu}^c - \bar{G}_{\mu\nu}^a D_\sigma^{ab} D_\sigma^{bc} G_{\mu\nu}^c) \\ &\quad - \frac{3}{8} m^2 \lambda_1 (\bar{B}_{\mu\nu}^a B_{\mu\nu}^a - \bar{G}_{\mu\nu}^a G_{\mu\nu}^a) + m^2 \frac{\lambda_3}{32} (\bar{B}_{\mu\nu}^a - B_{\mu\nu}^a)^2 \\ &\quad \left. + \frac{\lambda^{abcd}}{16} (\bar{B}_{\mu\nu}^a B_{\mu\nu}^b - \bar{G}_{\mu\nu}^a G_{\mu\nu}^b) (\bar{B}_{\rho\sigma}^c B_{\rho\sigma}^d - \bar{G}_{\rho\sigma}^c G_{\rho\sigma}^d) \right], \end{aligned} \quad (7)$$

which is still  $BRST_1$  invariant (but not  $BRST_2$ !), whereby the classical part  $S_{cl}$  is gauge invariant. For reasons of renormalizability, we had to introduce the scalar (mass) couplings  $\lambda_{1,3}$  and the gauge invariant tensor coupling  $\lambda^{abcd}$ . Notice that the new quartic interaction  $\propto \lambda^{abcd}$  in the novel fields spoil the unity. One might therefore question the equivalence with massless YM theories when  $m = 0$ . In [16], we have been able to show that

$$\langle \text{YM functional} \rangle_{S_{YM} + S_{gf}} = \langle \text{YM functional} \rangle_{S_{phys} + S_{gf}} \quad (8)$$

by making use of the  $\delta_s$ -cohomology, meaning that expectation values of “pure” YM functionals remain unchanged when calculated in our massless model. As a consequence, there will be no  $\lambda^{abcd}$  independence in those pure YM Green functions, and RG functions of the original YM quantities will remain unchanged, as long as massless renormalization schemes are employed. This fact was confirmed by explicit 1- and 2-loop computations in [14, 16]. In addition, also several other RG functions were computed. The obtained results were consistent with the Ward identities and confirmed the renormalizability explicitly. We end this section by mentioning that although the  $BRST_2$  symmetry is broken, with an associated broken Slavnov-Taylor identity  $ST_2$ , it is nevertheless possible to show that the ensuing Ward identities between a large class of Green functions are as if the  $ST_2$  would be unbroken [17].

### 3 Unitarity analysis

Since we have a massive gauge model with nilpotent  $BRST_1$  symmetry generator, we might hope that the theory would be unitary. Assuming that we start from the action, and that we take the elementary field excitations as asymptotic states, how can we prove the unitarity of the  $\mathcal{S}$ -matrix? This rather complicated task was performed in [18], whereto we refer for all details. One of the main tools was the use of the (free) BRST cohomology. We start from the free action  $S_0$  with free BRST symmetry  $s_0$  with nilpotent charge  $\mathcal{Q}_0$  ( $\mathcal{Q}_0^2 = 0$ ). Physical states  $|\psi_p\rangle$  are defined as belonging to the  $\mathcal{Q}_0$ -cohomology, i.e.

$$|\psi_p\rangle \in \mathcal{H}_{phys} \Leftrightarrow \mathcal{Q}_0|\psi_p\rangle = 0, \quad |\psi_p\rangle \neq |\dots\rangle + \mathcal{Q}_0|\dots\rangle, \quad \mathcal{Q}_0|\dots\rangle = 0. \quad (9)$$

Then the 2 remaining questions are: is this definition invariant under time evolution, described by the  $\mathcal{S}$ -matrix, and do the physical states have a positive norm, a condition sine qua non for a sensible quantum theory. Concerning the time evolution, we recall that in the operator language  $\mathcal{S} = \mathcal{T} \left[ e^{-i \int_{-\infty}^{+\infty} H_{int}(t) dt} \right]$ , so that  $\mathcal{S}|\psi_p\rangle \in \mathcal{H}_{phys}$ , if we require that  $[\mathcal{S}, \mathcal{Q}_0] = 0$ . We can pass to the path integral language to rephrase this condition into one for the action  $S$ , namely

$$s_0 e^{iS} \cong 0 \quad \text{on-shell, i.e. modulo free equations of motion.} \quad (10)$$

This equation can then be solved iteratively, in particular order by order in the available coupling constant(s).

$$\text{action } S = S_0 + S_1 + \dots, \quad \text{BRST } s = s_0 + s_1 + \dots,$$

while it is proven that

$$(s_0 + \dots + s_i)(S_0 + \dots + S_i) = 0, \quad (s_0 + \dots + s_i)^2 = 0 \text{ to } i^{th} \text{ order.} \quad (11)$$

If we are lucky, the procedure stops at finite order, and we end up with an action  $S$ , invariant under a BRST symmetry  $s$  with nilpotent generator  $\mathcal{Q}$ , with the desired property that the physical subspace  $\mathcal{H}_{phys}$  is invariant under time evolution. We used a “backward” argument: if we start from an action  $S$  with nilpotent symmetry charge  $\mathcal{Q}$ , then this is a solution of the previous procedure starting from the corresponding free counterparts  $S_0$  and  $\mathcal{Q}_0$ , obtained by switching of any couplings. Having answered the time evolution question, we are still left with the positive norm issue. We suffice by saying this required a rather lengthy and technical study. One of the problems faced was the occurrence of multipole fields. Nevertheless, a complete analysis was provided in [18], yielding a negative result: negative norm states do appear in the physical subsector of the theory, therefore it is not unitary. The reader might have

immediately questioned the wisdom of trying to prove the unitarity of this model, as it has its root in a nonlocal field theory, which are known to have problems with ghost states. Next to the presence of a nilpotent BRST invariance, there is however another reason why this endeavour was worth the effort. Specializing to the Abelian case, it can be shown that after integrating out the auxiliary fields, we obtain the Abelian Stueckelberg model, where the Stueckelberg scalar has been integrated out. We recall that the Abelian Stueckelberg model is renormalizable and unitary, see e.g. [19] for a review. If we would analyze the unitarity of the Abelian version of our model, we would run into exactly the same problem as in the non-Abelian case, i.e. the presence of negative norm states in the physical subspace. Apparently, the way of localizing a nonlocal action plays a substantial role.

## 4 Discussion

Since our action is perturbatively equivalent with YM in the massless case, we could take it as starting point instead of YM. As proven, we can couple mass terms to it, without ruining renormalization/gauge invariance requirements. The next challenge would of course be trying to construct a sensible gap equation to produce a dynamically generated value for  $m \propto \Lambda_{QCD}$ . Afterwards, we could start looking at potential nonperturbative  $\frac{m^2}{q^2}$  power corrections appearing in gauge (in)variant correlators. We are thus interested in nonperturbative effects in an asymptotically free theory, which occur in an energy region below the high energy (asymptotic) region where the elementary fields excitations are observables. Since this is YM ( $m \equiv 0$ ), there is no unitarity issue at high energies: 2 transverse gluon polarizations are physical. At lower energies, the gluons etc still are our effective degrees of freedom, but will behave like quasi particles, corrected by nonperturbative effects due to the generated  $m \neq 0$ . Lack of unitarity in terms of gluons is hence not a problem, but rather finding the correct physical degrees of freedom, which of course corresponds to the task of proving confinement.

## Acknowledgments.

D. Dudal is a Postdoctoral Fellow and N. Vandersickel a PhD Fellow of the Research Foundation-Flanders (FWO Vlaanderen). The Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq-Brazil), the SR2-UERJ and the Coordenação de Aperfeiçoamento de Pessoal de Nível Superior (CAPES) are gratefully acknowledged for financial support.

## References

- [1] K. Langfeld, H. Reinhardt and J. Gattnar, Nucl. Phys. B **621**, 131 (2002).
- [2] K. Amemiya and H. Suganuma, Phys. Rev. D **60**, 114509 (1999).
- [3] V. G. Bornyakov, M. N. Chernodub, F. V. Gubarev, S. M. Morozov and M. I. Polikarpov, Phys. Lett. B **559**, 214 (2003).
- [4] J. H. Field, Phys. Rev. D **66**, 013013 (2002).
- [5] J. M. Cornwall, Phys. Rev. D **26**, 1453 (1982).
- [6] A. C. Aguilar and A. A. Natale, JHEP **0408**, 057 (2004).
- [7] A. C. Aguilar and J. Papavassiliou, JHEP **0612**, 012 (2006).
- [8] H. Vershelde, K. Knecht, K. Van Acoleyen and M. Vanderkelen, Phys. Lett. B **516**, 307 (2001).
- [9] K. G. Chetyrkin, S. Narison and V. I. Zakharov, Nucl. Phys. B **550**, 353 (1999).
- [10] Ph. Boucaud, A. Le Yaouanc, J. P. Leroy, J. Micheli, O. Pene and J. Rodriguez-Quintero, Phys. Rev. D **63**, 114003 (2001).
- [11] O. Andreev, Phys. Rev. D **73**, 107901 (2006).
- [12] F. V. Gubarev and V. I. Zakharov, Phys. Lett. B **501**, 28 (2001).
- [13] D. Dudal, H. Vershelde and S. P. Sorella, Phys. Lett. B **555**, 126 (2003).
- [14] M. A. L. Capri, D. Dudal, J. A. Gracey, V. E. R. Lemes, R. F. Sobreiro, S. P. Sorella and H. Vershelde, Phys. Rev. D **72**, 105016 (2005).
- [15] R. Jackiw and S. Y. Pi, Phys. Lett. B **403**, 297 (1997).
- [16] M. A. L. Capri, D. Dudal, J. A. Gracey, V. E. R. Lemes, R. F. Sobreiro, S. P. Sorella and H. Vershelde, Phys. Rev. D **74**, 045008 (2006).
- [17] M. A. L. Capri, D. Dudal, V. E. R. Lemes, R. F. Sobreiro, S. P. Sorella, R. Thibes and H. Vershelde, Eur. Phys. J. C **52**, 459 (2007).
- [18] D. Dudal, N. Vandersickel and H. Vershelde, Phys. Rev. D **76**, 025006 (2007).
- [19] H. Ruegg and M. Ruiz-Altaba, Int. J. Mod. Phys. A **19**, 3265 (2004).